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A Magnetic Trap For Simultaneous Confinement of Neutral Atoms and a Non-Neutral Plasma

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Abstract.
Three methods have been proposed for the simultaneous confinement of neutral atoms and a non-neutral plasma in close proximity [D.H.E. Dubin, Phys. Plasmas **8**, 4331 (2001)]. This note discusses one of these methods, in which particles are trapped in an axially-symmetric static magnetic field with a magnetic minimum in a ring around the axis of symmetry. Axial symmetry is required for confinement of the rotating non-neutral plasma, and the magnetic minimum traps the neutral atoms. This trap design may be useful for the production and confinement of cold antihydrogen.

INTRODUCTION

Over the past decade, minimum **B** magnetic traps have developed into a standard technology for trapping neutral clouds of atoms [1]. These traps use magnetic fields with a local minimum in $|\mathbf{B}(\mathbf{r})|$ at a point where $|\mathbf{B}|$ remains nonzero.

In one standard trap design, the "Ioffe-Pritchard" trap [2], a pair of Helmholtz coils create a saddle point in $|\mathbf{B}|$ at the trap center. The saddle is transformed to a minimum by four Ioffe bars. The magnetic minimum creates a potential well for some atoms since atomic spins μ align either with or against the magnetic field. The magnetic dipole energy for spins aligned against the field is

$$E_M = -\mu \cdot \mathbf{B} = |\mu||\mathbf{B}|, \tag{1}$$

which is minimized at the trap center. The magnetic field does not vanish in the well; otherwise a spin could flip at the magnetic null ("Majorana flips" [3]) and the magnetic minimum would then expel the atoms rather than confine them.

The Ioffe-Pritchard trap is an excellent configuration for many purposes. However, the design is not cylindrically symmetric, and this is a problem for certain applications. One such application is the production and confinement of cold antihydrogen. Two experiments are currently underway to achieve this goal [4]. The experiments use Penning traps to trap two cold single species non-neutral plasmas consisting of anti-protons and positrons respectively, from which it is hoped that antihydrogen can be created by careful combination of the two species. The antihydrogen must then be confined in a neutral atom trap. A recent analysis [5] has shown that stable single-particle orbits exist for individual positrons and antiprotons trapped in the non-axisymmetric magnetic field produced by a Ioffe-Pritchard trap. However, if the trapped single-species plasmas are cold and of high density so as to maximize recombination rates when they are combined,

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the large-scale static field asymmetries produced by the Ioffe bars will degrade plasma confinement, causing plasma heating and charged particle loss at levels that are probably unacceptable [6].

In recent work [7] three possible methods were put forward for trapping neutral atoms in close proximity to a cold, dense non-neutral plasma. One method relies on a novel cylindrically-symmetric magnetic configuration that can be used as a Penning trap to confine a non-neutral plasma, but also has a minimum in $|\mathbf{B}|$ so that it can simultaneously trap neutral atoms. The magnetic minimum is on a ring around the axis of symmetry of the trap. Potential well depths of order 1°K or more should be achievable with this trap design. It is important for the well to be as deep as possible, since the antihydrogen will be created at the plasma temperature of a few °K, and will also have kinetic energy associated with the plasma rotation. In the next section we consider this trap design in more detail.

A second design, discussed in Ref. 1, uses a standard "time-averaged orbiting potential" (TOP) trap [8] to confine the neutral atoms. Like the Ioffe-Pritchard trap, this design imposes large-scale azimuthal magnetic asymmetries, but the asymmetries are made to rotate around the trap axis. This rotating magnetic field has two effects: first, the torque exerted on the plasma by the rotating field may act to spin up the plasma, possibly even keeping it confined indefinitely in much the same manner as electrostatic "rotating wall" asymmetries used in other experiments [9]. Second, the rotating field, in concert with a static cusp field, creates a time-averaged magnetic minium at the trap center, with $|\mathbf{B}| \neq 0$ there. However, the effective depth of this potential minimum is probably limited to of order 0.1°K or less.

Reference 1 also considers a third design that, like the first design, uses static cylindrically-symmetric magnetic fields to confine both the plasma and the neutral atoms. Here the magnetic minimum is a null at the trap center. In order to keep neutral atoms away from this magnetic null, the neutral atom cloud is made to spin, creating a centrifugal potential that repels the cloud of neutrals from the trap center. The rate of rotation may be controlled with a small rotating magnetic field. The main static magnetic field contains sufficiently high multipole moments ($n = 4$ or higher) in order to overcome the centrifugal potential at large radius and trap the atoms in a ring. In this design, well depths of order 1°K or more could be obtained, depending on the rotation frequency of the neutral atoms and the strength of the magnetic field.

Although antihydrogen formation requires the recombination of positron and antiproton plasmas, the work presented here considers only the trapping of a single species plasma in conjunction with a neutral atom trap. There are several reasons for this: first, separate single species plasmas must be confined and cooled before recombination can be attempted. It would be easiest to do this nearby to the region where recombination and neutral atom trapping will occur, so it is important to consider the equilibrium of single species plasmas in the fields created by a neutral atom trap. Second, the recombination process itself is not yet understood: the partially-neutralized plasma may exhibit a host of instabilities, and issues regarding both the axial and radial confinement of such plasmas have not yet been resolved, although progress is being made [10, 11, 12]. By focussing on the trapping of a single species plasma in conjunction with neutral atoms, our work avoids these thorny issues.

However, our work could have some bearing on the recombination process: one

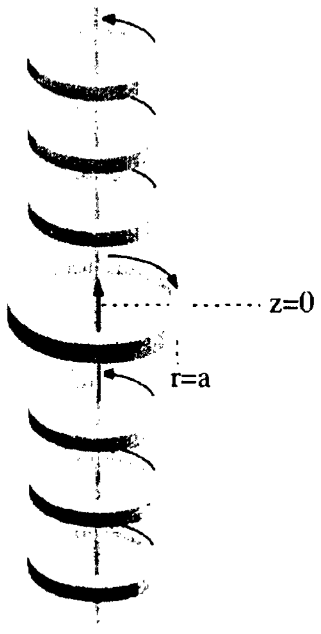

FIGURE 1. Magnetic coil configuration of a cylindrically-symmetric minimum-B Penning trap.

possible recombination technique would be to allow only a relatively small number of antiprotons at a time to enter into the positron region, so that the positron plasma remains nearly completely unneutralized. It might be hoped that a small density of antiprotons would have an insignificant effect on the positron stability, and that a description of the system as a single species plasma might then be relevant. In this scenario, one of the traps discussed in this paper could serve as the recombination section.

A CYLINDRICALLY-SYMMETRIC MINIMUM-B PENNING TRAP

Cylindrically-symmetric minimum-B equilibria have received considerable attention in the neutral plasma community due to their superior MHD stability properties [13]. The design considered here is similar to the "stuffed cusp," but with an added solenoidal magnetic field, which is required for non-neutral plasma confinement.

A schematic of the magnetic elements in the trap is shown in Fig. 1. The trap consists of a solenoidal magnetic field $B_0\hat{z}$, which for simplicity is assumed to be uniform; an azimuthal magnetic field $B_\theta\, a\hat{\theta}/r$ created by a wire aligned along the axis of the solenoid, and the field from a current loop concentric with the solenoid whose current runs in the opposite direction to that of the solenoid. The latter field $\mathbf{B}_\ell(r,z)$ is most easily written

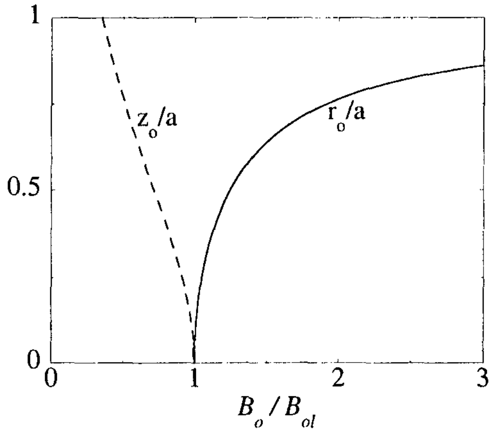

FIGURE 2. r and z location of the field null as the ratio of the solenoidal to loop field is varied. No B_θ field is applied.

in terms of the vector potential associated with the loop, assumed to have radius a:

$$\mathbf{B}_\ell(r,z) = -\frac{\partial A_{\theta_\ell}}{\partial z}\hat{r} + \frac{1}{r}\frac{\partial}{\partial r}(rA_{\theta_\ell})\hat{z}, \tag{2}$$

where

$$A_{\theta_\ell}(r,z) = -\frac{2B_{0_\ell}a^2}{\pi\sqrt{a^2+r^2+z^2+2ar}}\left[\frac{(2-m)K(m)-2E(m)}{m}\right], \tag{3}$$

$$m = \frac{4ar}{a^2+r^2+z^2+2ar}, \tag{4}$$

$K(m)$ and $E(m)$ are the complete elliptic functions [14], and $B_{0_\ell} = B_l(0,0)$.

When $B_{0_\ell} < B_0$, the loop magnetic field cancels out the solenoidal field at a ring of radius r_0 in the plane of the loop (the $x-y$ plane). When $B_{0_\ell} > B_0$, the field is cancelled at points $\pm z_0$ along the z axis. The dependence of r_0 and z_0 on B_0/B_{0_ℓ} is shown in Fig. 2.

Without an applied azimuthal field B_θ, the ring at $r = r_0$ and $z = z_0$ is both a magnetic minimum and a magnetic null, which makes it unusable as an atom trap. With an azimuthal field, $|\mathbf{B}|$ no longer vanishes. Since the azimuthal field is monotonically decreasing with r, the location of the minimum is shifted outwards to a location $r_m > r_0$. For B_θ sufficiently large, the minimum disappears as it is pushed toward the loop.

Contours of constant $|\mathbf{B}|$ are shown in grayscale in Fig. 3 for the choices $B_0/B_{0_\ell} = 2.6$, giving $r_0 = 0.81a$; and $B_\theta/B_0 = 0.125$, giving $r_m = 0.83a$. For these parameters, contours of constant $|\mathbf{B}|$ with $|\mathbf{B}|/B_0 \lesssim 0.9$ do not intersect the walls of the apparatus.

For antihydrogen in the ground state, $|\mu| \simeq \hbar e/2m_e c$. According to Eq. (1), a solenoidal field of $B_0 = 2$ T therefore corresponds to a potential well of depth $(0.9 - 0.125/0.83)\mu B_0 = 1°$K. Atoms with temperature less than this should collect around the minimum at $r = r_m$. A field $B_0 \gtrsim 8$ T increases the well depth above the temperature of liquid helium, obviating the need to cool the trap walls with a dilution refrigerator.

We now turn to the non-neutral plasma confinement characteristics of this trap design. It is well-known that single-species plasmas can be confined in a thermal equilibrium state by static cylindrically symmetric fields [15]. The unneutralized charge cloud rotates about the trap axis with rotation frequency ω. For low plasma temperature such that the Debye length is small compared to the plasma size, the plasma density in thermal equilibrium is determined by the equation [15]

$$n(r,z) = \frac{m\omega}{2\pi e^2}(\Omega_{c_z}(r,z) - \omega), \tag{5}$$

where $\Omega_{c_z} = eB_z(r,z)/mc$ is the cyclotron frequency based only on the axial component of the total magnetic field. Here e is the charge and m the mass of the plasma particles. Note that the azimuthal field B_θ does not play a role in non-neutral plasma confinement.

Equation (5) follows from the fact that rotation through a magnetic field creates an effective potential well $\phi_B(r,z)$ for the particles, where

$$\phi_B(r,z) = \frac{e\omega r}{c}A_\theta(r,z) - \frac{m\omega^2 r^2}{2}. \tag{6}$$

The second term is the deconfining centrifugal potential and $A_\theta(r,z)$ is the θ-component of the magnetic vector potential, given by

$$A_\theta(r,z) = A_{\theta_\ell}(r,z) + \frac{1}{2}B_0 r^2, \tag{7}$$

where A_{θ_ℓ} is the loop vector potential, given by Eq. (3). The potential ϕ_B can be thought of as due to a fictitious neutralizing background charge of density $n_B(r,z)$; that is, $\nabla^2\phi_B(r,z) = 4\pi e^2 n_B(r,z)$. The equilibrium plasma density matches the density n_B out to a surface of revolution where the supply of plasma charge is exhausted. The shape of this surface is determined by the condition that it is an equipotential in the frame rotating with the plasma. Therefore to find the plasma shape, we solve the equation

$$\phi(r,z)|_S = \text{constant} \tag{8}$$

where S is the surface of the plasma, and ϕ is the total potential, including the self-consistent potential $\phi_P(r,z)$ from the plasma itself, and the potential $\phi_V(r,z)$ from voltages on surrounding electrodes:

$$\phi(r,z) = \phi_P(r,z) + \phi_V(r,z) + \phi_B(r,z). \tag{9}$$

Since our design has a central wire, required for the azimuthal field B_θ, a voltage V must be applied to the wire in order to repel plasma charges. This creates a hollow plasma. In Fig. 3 we assume the wire has radius $b = 0.05a$, and that there is a surrounding grounded cylindrical electrode at $r = a$. The plasma is assumed to be a long column. For large $|z|$ away from the loop, the magnetic field is nearly uniform, and the plasma forms a hollow column with uniform density $n_0 = (m\omega/2\pi e^2)(\Omega_0 - \omega)$, where $\Omega_0 = (eB_0/mc)$ is the

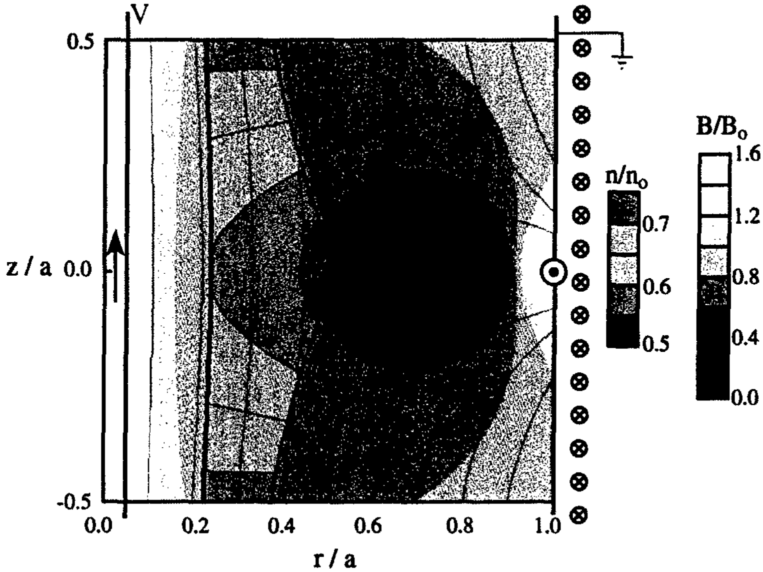

FIGURE 3. An example of plasma confinement in the cylindrically-symmetric minimum-B Penning trap. Plasma density is shown by colored contours, magnetic field intensity is shown by gray-scale contours. Green lines are magnetic flux surfaces. Arrows denote the location of current-carrying wires. Thick blue lines are electrodes. In this example, $B_0/B_{0_\ell} = 2.6$, $B_\theta/B_0 = 0.125$, and $V = 0.38\pi e n_0 a^2$.

cyclotron frequency associated with the solenoidal field only. The inner and outer radii of the hollow column, r_1 and r_2, are related to the voltage V on the inner wire by the condition that $\phi(r_1,z) = \phi(r_2,z) = \text{constant}$:

$$V = \pi e n_0 \left[r_2^2 - r_1^2 + 2r_1^2 \ln(\frac{r_1}{b}) - 2r_2^2 \ln(\frac{r_2}{a})\right]. \tag{10}$$

In Fig. 3, we have chosen $r_1 = 0.2a$, $r_2 = 0.3a$, so that $V = 0.38\pi e n_0 a^2$. We have also assumed that $\omega/\Omega_0 \ll 1$, so that we can neglect the centrifugal potential in ϕ_B [see Eq. (6)]. In this case we need not specify ω in determining the plasma equilibrium.

One can see from the figure that the plasma expands radially and decreases in density near $z = 0$, since the confining B_z field is weakest here. One can also see that the minimum B ring is well outside the plasma. This is hardly surprising, since B_z vanishes at $r_0 < r_m$, and the plasma must be confined in a region away from this point according to Eq. (5). In principle, it is possible to construct finite-length plasma equilibria that contain the minimum ring at r_m since B_z does not vanish at r_m, but rather at r_0. However, the confinement potential $\phi(r,z)$ exhibits only a very weak minimum in this case.

Since the plasma rotates with frequency ω, neutral atoms created by recombination at radius r within this plasma will be created with a certain amount of angular momentum, $\ell_z = M v_\theta r$, where M is the atomic mass and v_θ is the azimuthal velocity, which has an average value of ωr). This will create a tendency for the neutral atom cloud to spin. This rotation creates a deconfining centrifugal potential for the neutral atoms, which can be expressed as an additional term in Eq. (1):

$$E_M = |\mu||\mathbf{B}| + \frac{1}{2}\frac{\ell_z^2}{Mr^2}. \tag{11}$$

So far we have assumed that this centrifugal term is negligible. However, this depends on the size of ℓ_z. In typical experiments, the rotation frequency $\omega/2\pi$ is on the order of several kHz. Taking the plasma radius to be $r_p \simeq 1$ cm, we may estimate angular momentum due to rotation as $\ell_z \sim M\omega r_p^2$. Then for an antihydrogen atom the centrifugal term is

$$0.5M\omega^2 r_p^4/r^2 = 0.24°\mathrm{K}\left(\frac{\omega/2\pi}{1\,\mathrm{kH_z}}\right)^2\left(\frac{r_p}{r}\right)^2\left(\frac{r_p}{1\,\mathrm{cm}}\right)^2.$$

One can see that a rotation frequency of 1 kHz and a plasma radius of 1 cm leads to a small centrifugal correction to E_M; however, $\omega = 10$ kHz would cause a large change in the magnetic well, possibly leading to deconfinement, depending on the ratio of r_p/r at the location of minimum B. In order to trap large densities required for rapid recombination, a large magnetic field will therefore be required so that ω remains small.

Another potential difficulty with this design involves the central wire. Astute readers may already have noted that producing a tesla size field with a central wire (or wires) requires exceedingly high currents. Fortunately, the azimuthal field produced by the central wires need not be this large. Recall that the only purpose of the azimuthal field is to prevent Majorana flips by keeping $|\mathbf{B}|$ finite at the magnetic minimum. For cryogenic atoms, this can be accomplished with modest azimuthal fields, on the order of a few gauss. The argument is as follows. Majorana flips are prevented when the minimum spin precession frequency, $2\mu B_{\mathrm{min}}/\hbar$ (where $B_{\mathrm{min}} = B(r_m, z_m)$ is the minimum magnetic field strength), is much greater than the maximum rate of variation of the magnetic field as seen by an atom moving through the minimum, $(\bar{v}\nabla B/B)_{\mathrm{max}} \sim \bar{v}B_0/aB_{\mathrm{min}}$, where $\bar{v}$ is the thermal speed of the atoms. Comparing these two rates for antihydrogen yields

$$B_{\mathrm{min}} \gg 1.3\,\mathrm{Gauss}\,(B_0/1\,\mathrm{Tesla})^{1/2}(T/1°K)^{1/2}(a/1\,\mathrm{cm})^{-1/2}. \tag{12}$$

Thus, a field of 10–100 Gauss at the magnetic minimum should be sufficient to prevent any Majorana flips from occuring, provided that the atoms are cold.

Finally, we note that it is possible to create a ring-minimum-B configuration that does not need a central wire: the "Furth-Andreoletti" trap [13]. This considerably simplifies the trap design, but unfortunately the depth of the magnetic minimum in such traps is exceedingly weak [16]. In Ref. 7 we discuss an axisymmetric trap design that also avoids the central wire, but has a well depth of order 1°K or more.

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